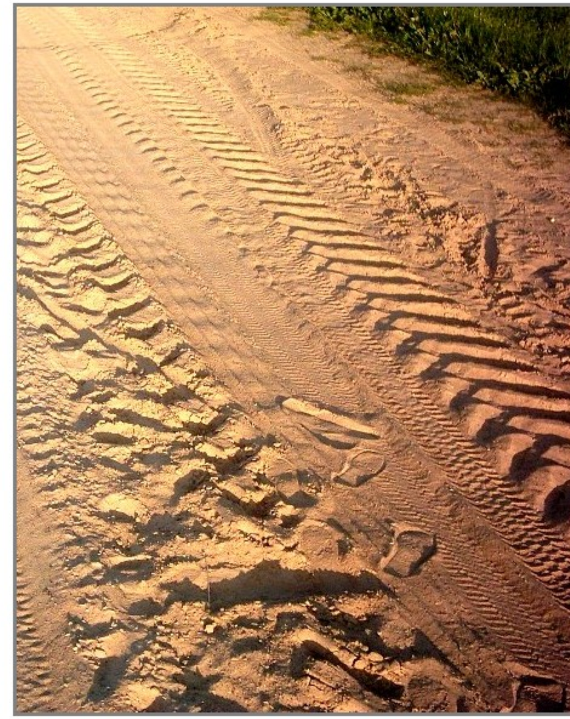


9th Austrian Young Physicists' Tournament' 2007

Team of Russia

9th Austrian Young Physicists' Tournament' 2007

Problem №14 «Earthquake»

Reporter: Alexander Kanivets,
Team of Russia

May 5, 2007

Problem №14 «Earthquake»

Suggest a mechanism that makes buildings resistant to earthquakes. Perform experiments and explain the results.

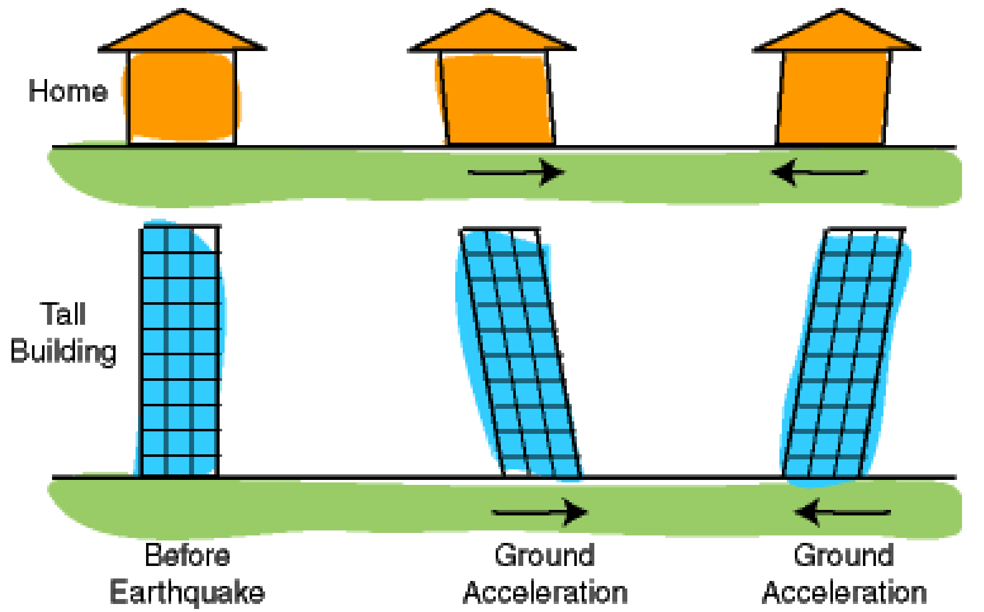
Problem formalization

What is seismic-resistance?

- Absolute resistance?
- Repairable damage?
- Undamaged building structure?

Problem formalization

- Ground motion along different axis (x, y, z)?
- Rotational motion?
- For each axis the following parameters can be measured
  - Shift value or amplitude
  - Velocity
  - Acceleration



Why earthquakes are dangerous for buildings?

- The building is deformable as any rigid body;
- There are 5 main deformation types: compression, stretch, shear, torsion, bending
- Building has different degrees of freedom (ways of oscillation).
- Building resonance is possible.

Weak ground floor is often the cause of destruction

Monolith structure is more resistant to ground oscillations than exterior walls

Pics: www.nationalgeographic.com\forcesofnature\earthquake\

Bearing construction - frame

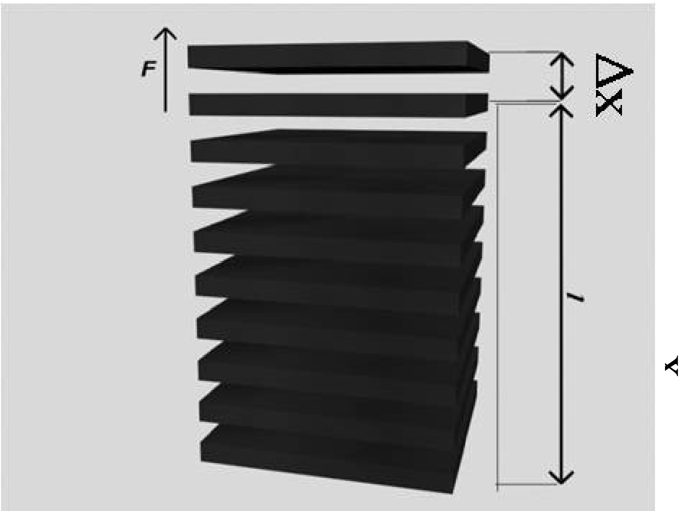
Rigid elements: vertical walls and horizontal plates

Forced oscillation of building unit and of building as a whole

- How many degrees of freedom?
- How many ways of oscillation?
- How many oscillation modes?
- Can a resonance occur?
- How can shear deformations be described?

Cookies are fixed with glue

Building deformation: compression and stretching



Deformation:

$$\varepsilon = \frac{\Delta x}{x} = \frac{\Delta \ell}{\ell}$$

where Δl is length increase.

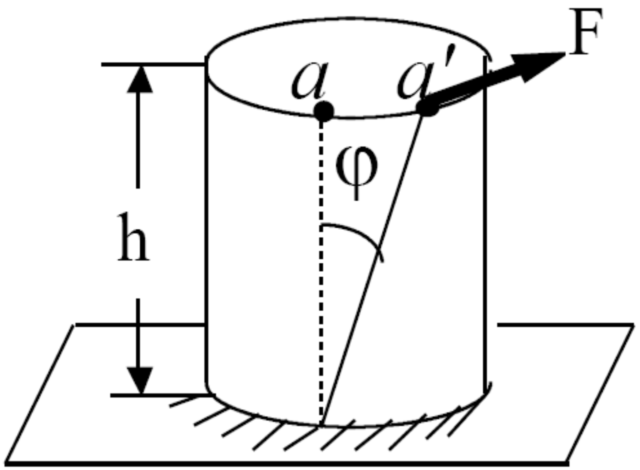
According to the Hook's law, stress is:

$$\sigma = \mathrm{E}\,\varepsilon = \frac{F}{S} = E\,\frac{\Delta \ell}{\ell}$$

where E is Young modulus.

Structures subjected to compression and stretching need to be made of elastic materials

Building deformation: torsion

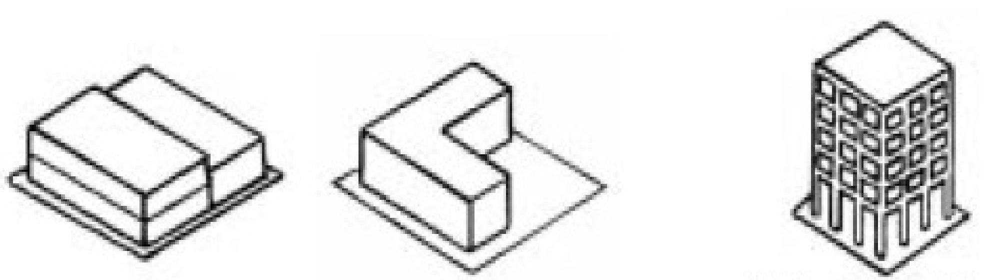


Torsional deformation:

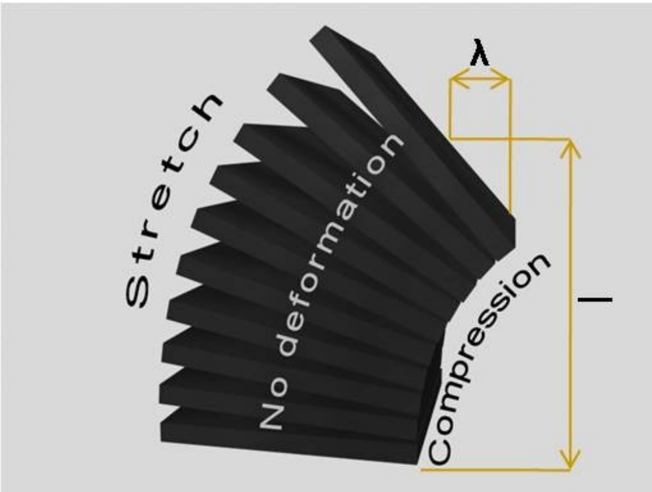
$$\varepsilon = \operatorname{tg}\varphi$$

$$\sigma = E\,\varepsilon$$

Building needs to be symmetrical relatively to the center of mass for inertia momentum equality.



Building deformation: bending



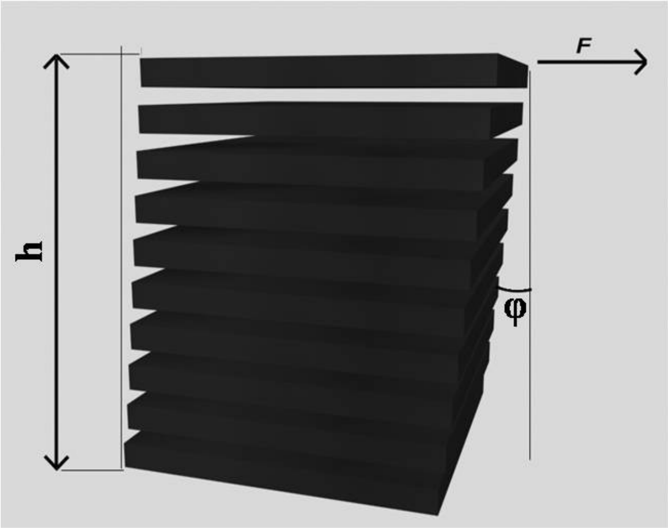
Relative deformation:

$$\varepsilon = \frac{\lambda}{\ell}$$

λ is bending arrow, l is initial length.

Deformation in middle part is negligible so structures subjected to bending can be hollow to decrease their mass.

Building deformation: shear



Shear deformation:

$$\varepsilon = \mathrm{tg}\varphi = \frac{\Delta x}{h}$$

Stress in building:

$$\sigma = E\,\varepsilon$$

Experimental model: theory of similarity

The following parameters need to be considered, to correctly reproduce real processes during experiments:

1. Material (proportionality of occurring stresses and fragility of construction)
2. Earthquake parameters (amplitude, shift direction, frequency of oscillation)

If the phenomenon S is defined by system of values $\{Y_\alpha\}$, then S_1 and S_2 are similar if system of values S_1 is equal to S_2, multiplied by the similarity coefficient:

$$Y_{\alpha 1} = kY_{\alpha 2}$$

Similarity principle
(“single shift” earthquake)

- Building deformation is defined by shift modulus:

$$G = \frac{\tau}{\gamma}$$

- where: $G = \frac{E}{2(1+\mu)}$, but Poisson ratio of concrete is very small so that:

$$\frac{\tau}{\gamma} = \frac{E}{2}$$

- Building failure starts at low stories, as the stress there is maximal and reaches:

$$\tau = \frac{ma}{S}, m\ is\ building\ mass,\ a\ is\ acceleration,\ S\ is\ walls\ area$$

- The exceeding of fragility limit or limit deformation angle can also become the cause of failure, but the deformation angle is scale invariant

Similarity principle

- The stress change evaluation after L times rescaling

$$\left.\begin{array}{l} m \propto L^3, S \propto L^2 \\ \tau \propto E \end{array}\right\} \Rightarrow E \propto L$$

- In order to make our model correct:
  - Failure modeling – breaking limit should be proportional to the size
  - Deformation modeling – Young's modulus should be proportional to the size

- Breaking limit of concrete is 10 – 40 MPa (depends on the brand 100, 150, 200, 400)
- Breaking limit of a «Mariya» cookie is 130 kPa
- Cookie building of 10 cm height is similar to concrete building of 30 meters height
- Breaking limit of glue is 40 kPa, breaking limit of cement is 2,5 MPa

Calculation of reduced Richter magnitude for our model

By using equation for potential energy per unit wavelength:

$$E_{P\lambda} = \frac{A^2 \rho \omega^2}{4}$$

$$\frac{E_{P_{\lambda}1}}{E_{P_{\lambda}2}} \equiv \frac{M_{\text{buiding}}}{M_{\text{model}}}$$

	Earthquake	Model
Amplitude, m	1	0,02
Density of media, kg/m³	100	20
Radian frequency, rad/s	62,8	25,12

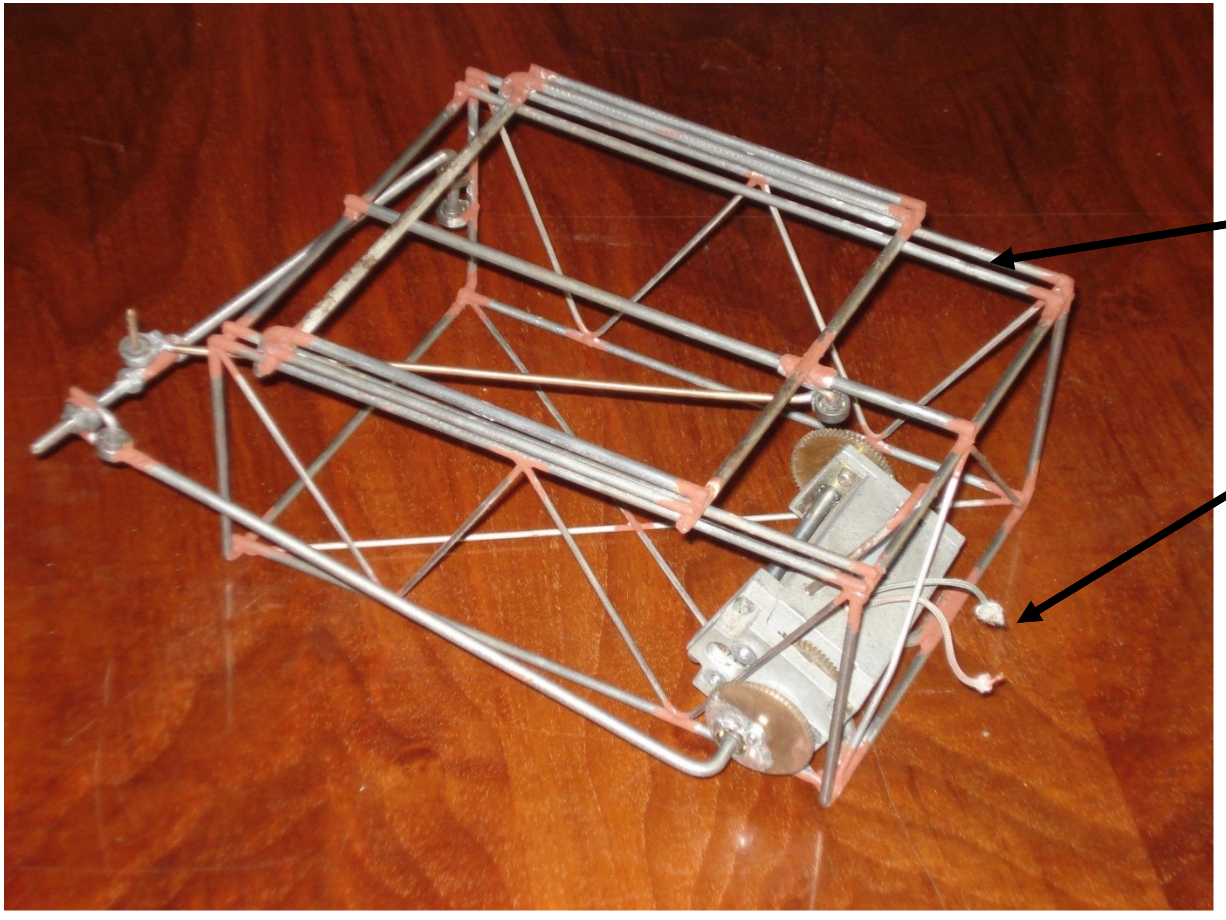
And substituting values of amplitude (A), density of media (ρ) and radian frequency (ω) for earthquake and experimental model, we obtain:

$$\frac{M_{\text{building}}}{M_{\text{model}}} \approx \frac{10^6 kg}{0.3kg} \approx 3 \cdot 10^6$$

Table values correspond to Kalamato earthquake, Greece,1989 with 7.9 Richter magnitude.

For model with mass ≈300 g «earthquake» has magnitude about 4.9.

Experimental setup developed by us



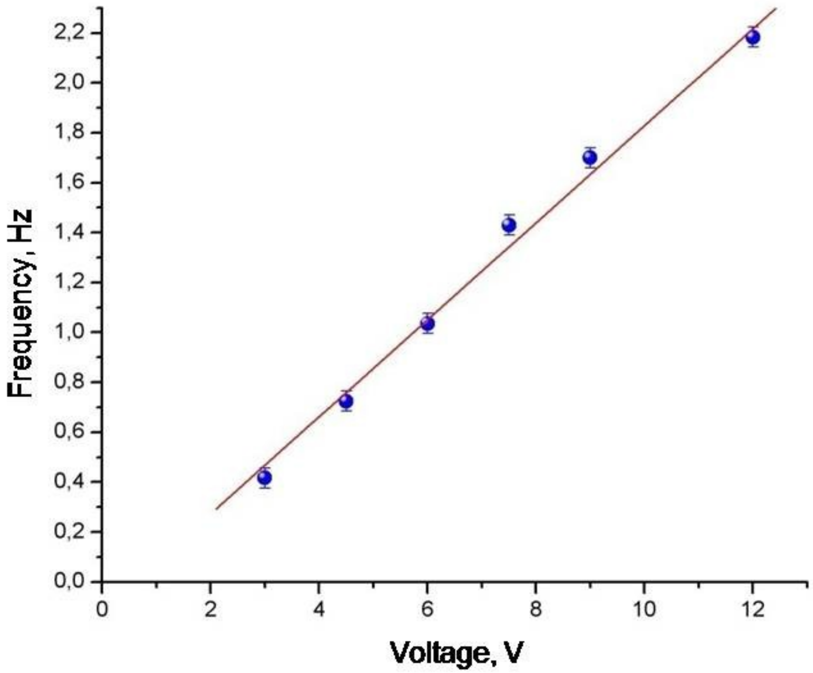
Mobile frame

motor

Setup scheme

Experimental setup, developed by us

- **Amplitude is varied from 2 to 4 cm;**
- **Frequency of oscillation is varied from 1.1 to 3.6 Hz;**
- **Setup is able to move objects up to 1 kg weight**



The setup allows to simulate oscillations in horizontal plane.

Conclusion: this setup allows to model reproducible "earthquakes" with defined parameters corresponding to similarity criteria.

Destruction of building with fragile frame

Glued cookies model:

Story size: 10x10x8 cm

Model allows to research the behavior of real uniform concrete building with different height.

Two stories:

Movie1

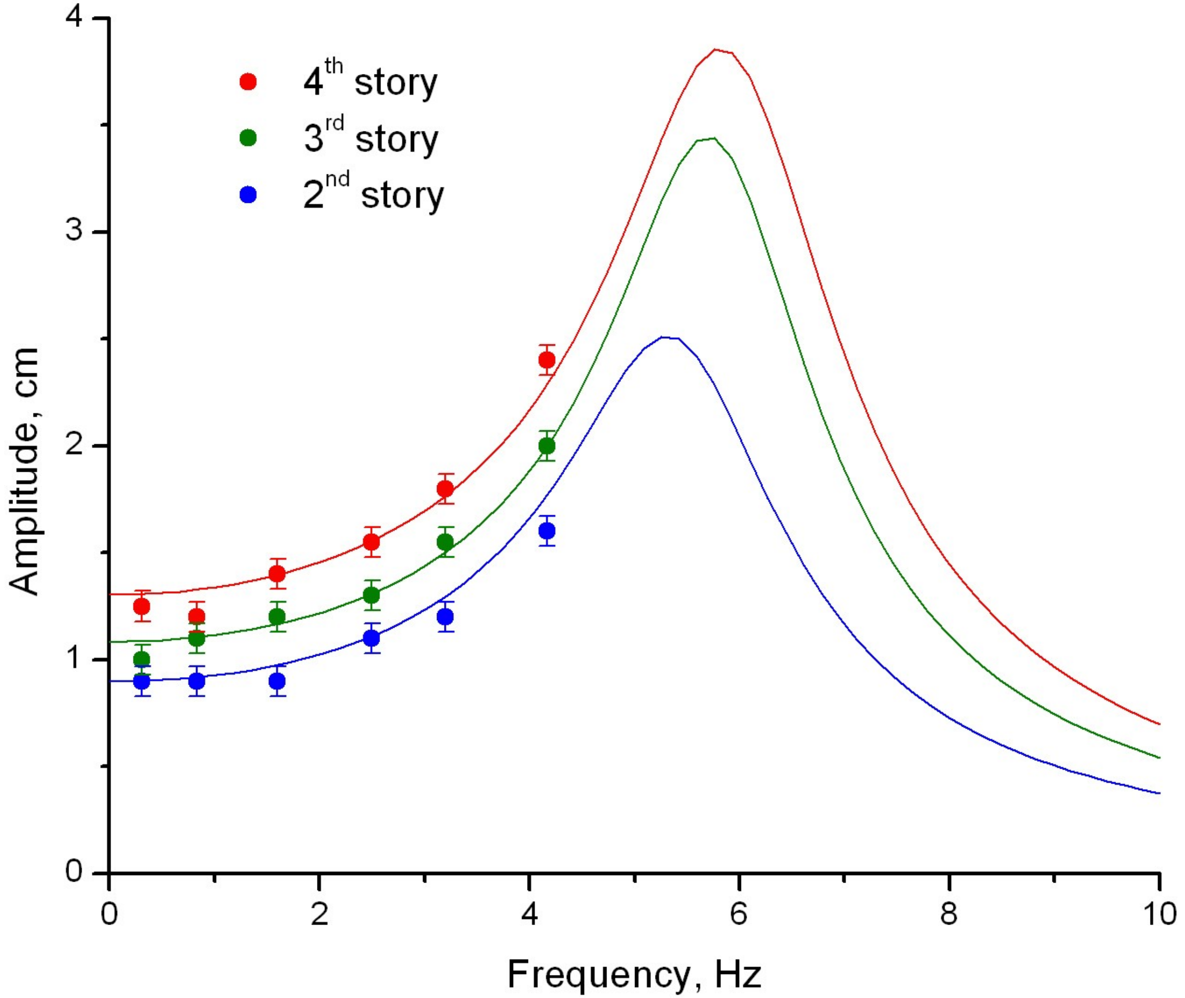
Model is demolished after exceeding fragility limit in joints.

Four stories: Movie3

Eight stories: Movie4

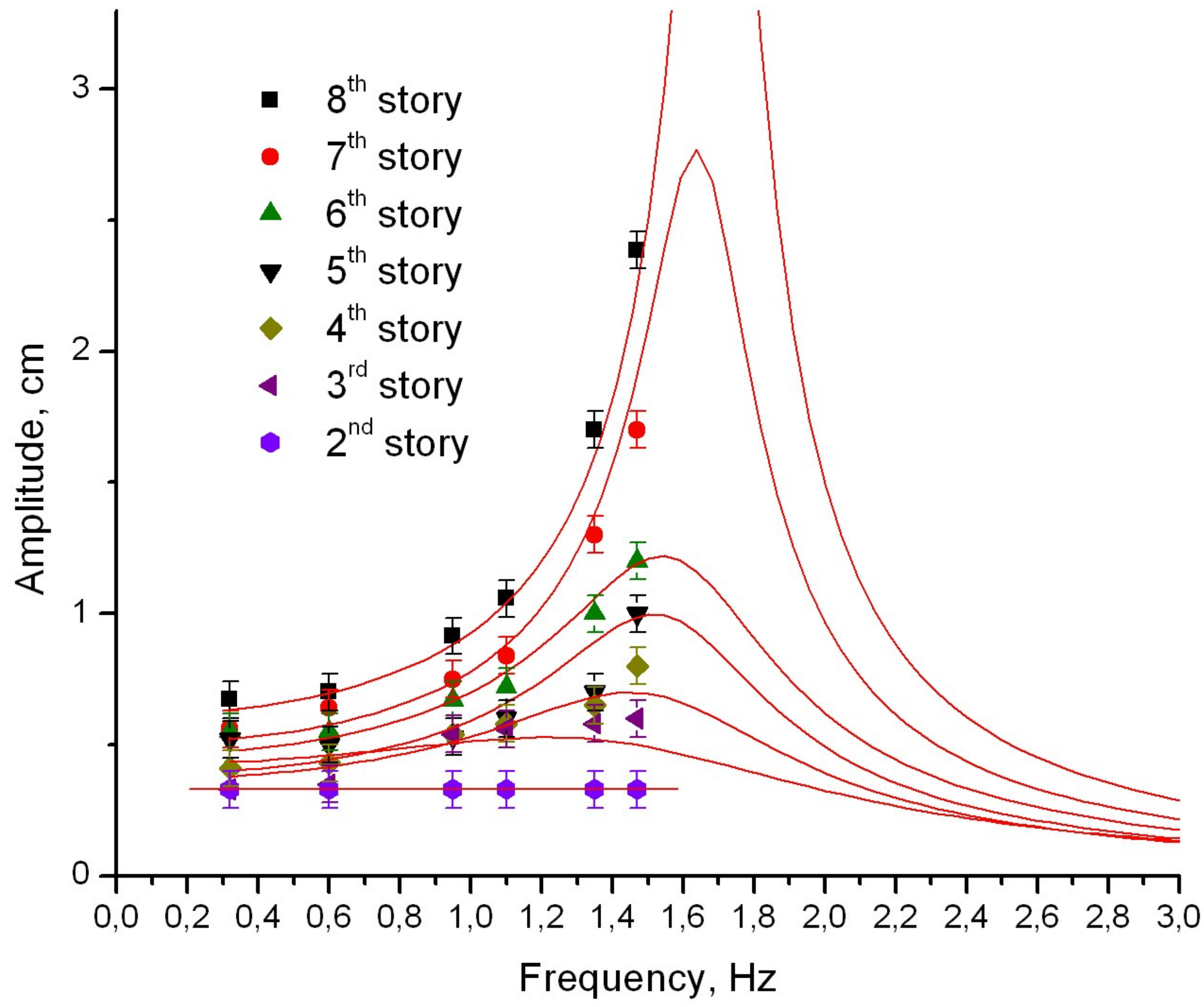
The resonance and avalanche-like destruction can be observed

Resonance curves for cookie building, 4 stories

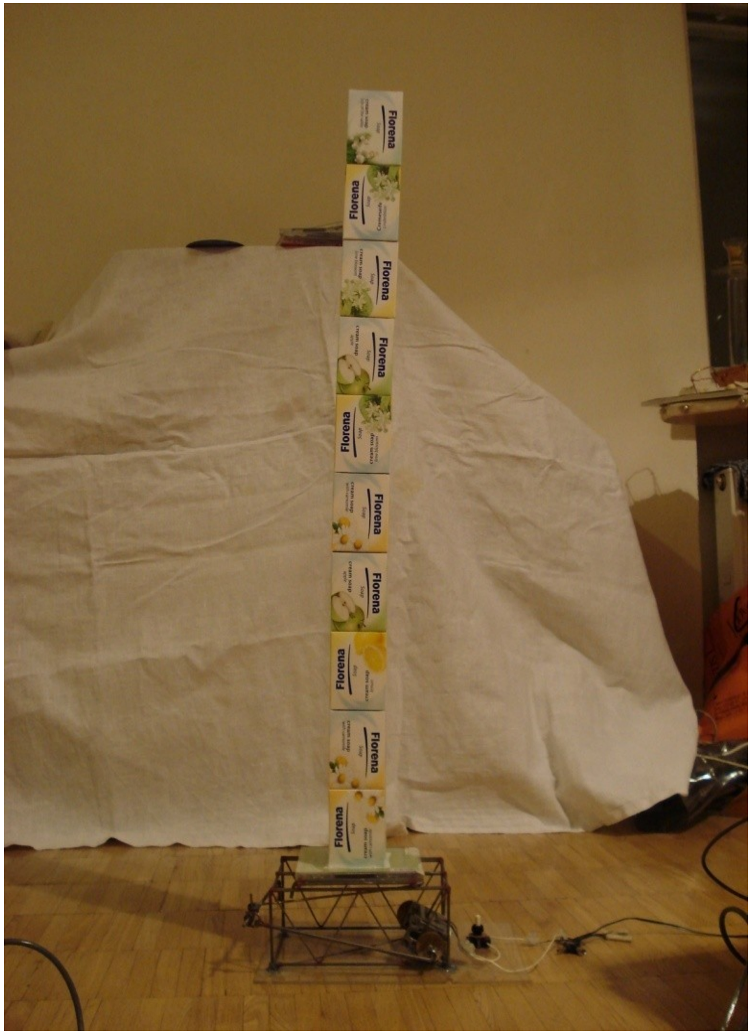


Theoretical curves on the picture fit well experimental data

Resonance curves for cookie building, 8 stories



Oscillation of building with elastic frame



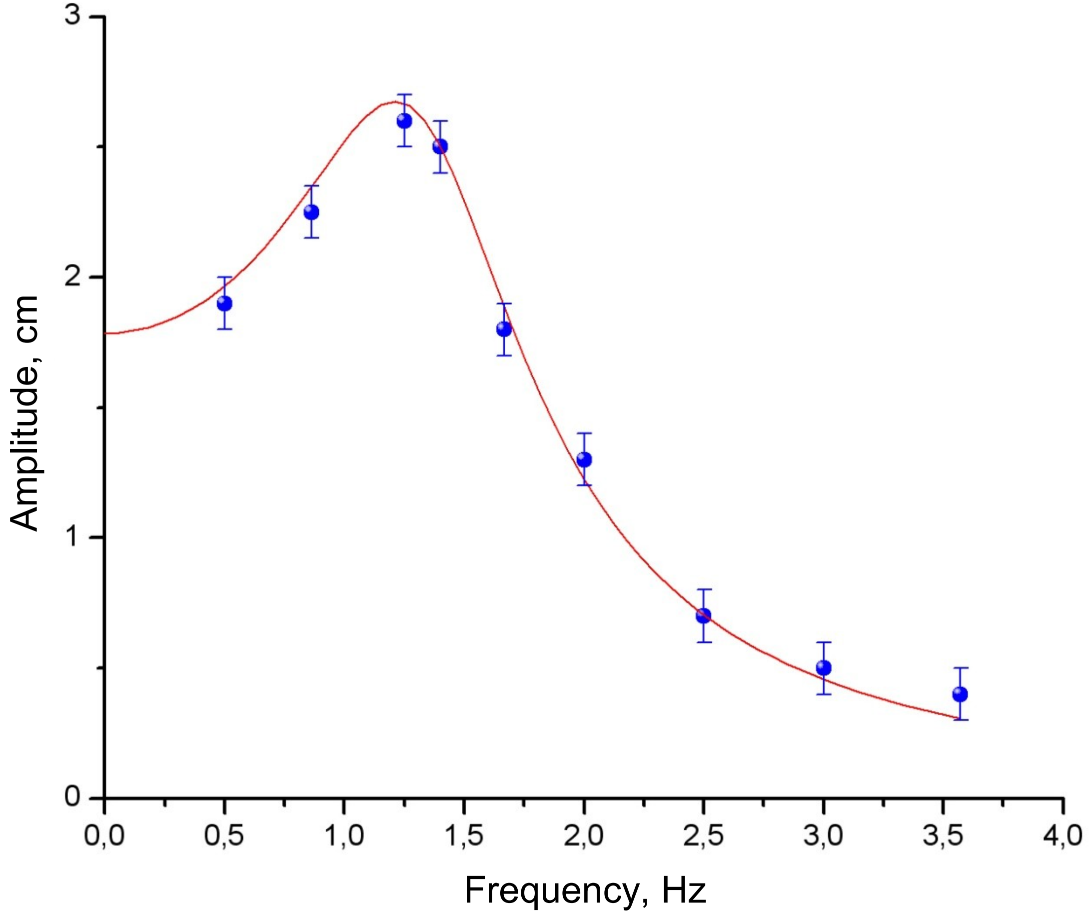
Model (87 cm height) was made of paper boxes glued with «Moment®».

Paper and glued joints have very high plasticity limit, «stories» are mobile relatively to each other and failure doesn't occur even under big deformations.

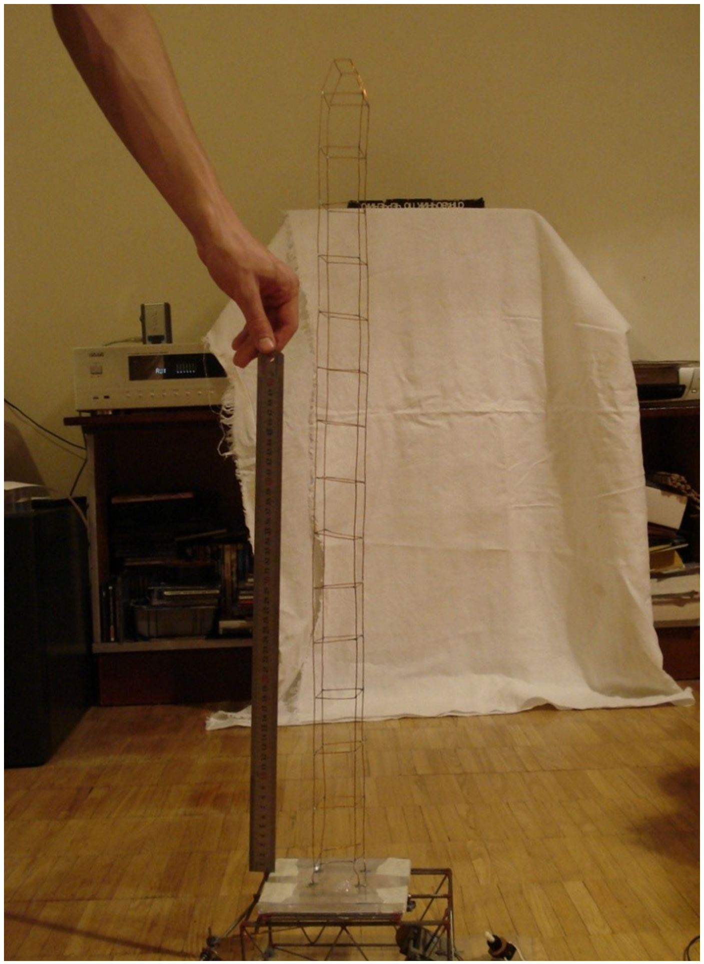
Conclusion: mobile exterior walls increase seismic-resistance.

Movie1 Movie2

Resonance curve for elastic building



Destruction of building with elastic metal wireframe



Building model (80 cm height) made of wire doesn’t fail due to high elasticity limit.

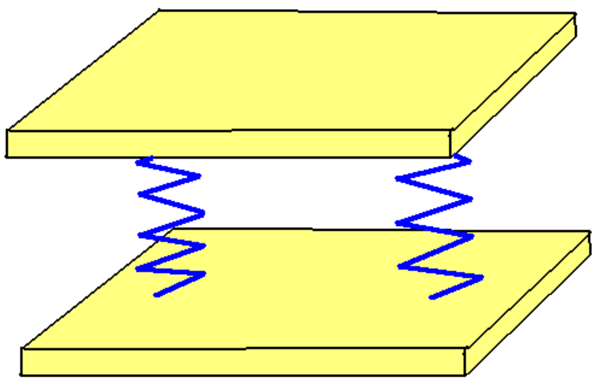
But: in elastic structures, different degrees of freedom can be observed.

- 1st degree of freedom
- 2nd degree of freedom
- 3rd degree of freedom

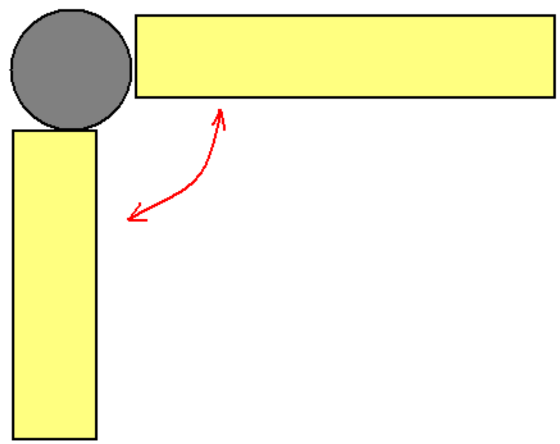
Measured parameters for experimental builing models

- Elastic building model:
  - Shear $k = 43$ N/rad;
  - Compression $\sigma = 10$ N/cm;
- Building model with metal wireframe
  - Shear $k = 248$ N/rad;
  - Compression $\sigma = 100$ N/mm;
- Cookie building model
  - Shear $k = 120$ N/rad;
  - Breaking point: 130 kPa

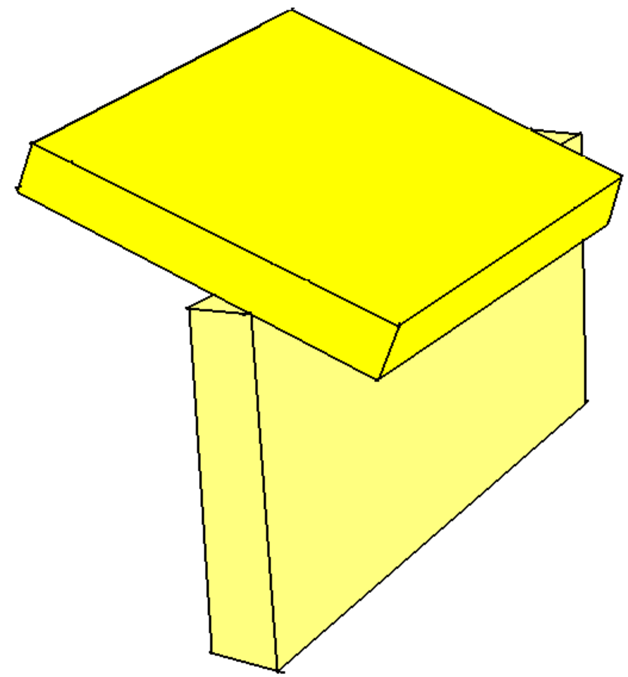
Building joints deformation model



Elastic connection between stories

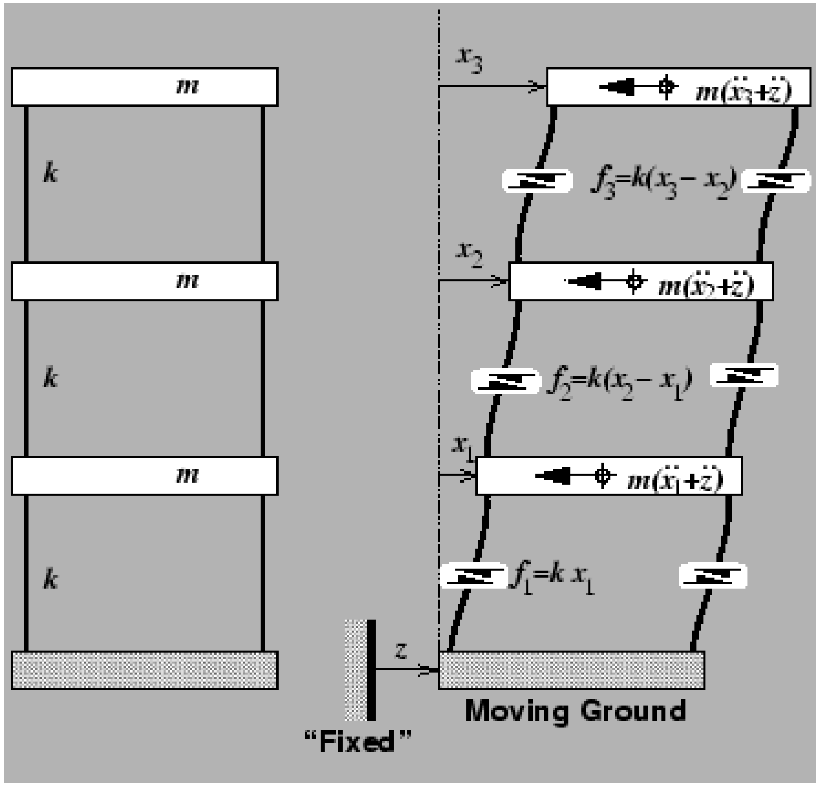


Elastic joint and rigid plates



Relative displacement of rigid unconnected plates

Theoretical model of building oscillation. Stresses at different stories



When one story is moving along another, the shear force occurs in the walls:

$$f = kx = ma$$

$$m\ddot{x}_1(t) + 2kx_1(t) - kx_2(t) = -m\ddot{z}(t)$$

$$m\ddot{x}_2(t) - kx_1(t) + 2kx_2(t) - kx_3(t) = m\ddot{z}(t)$$

$$m\ddot{x}_3(t) - kx_2(t) + kx_3(t) = -m\ddot{z}(t)$$

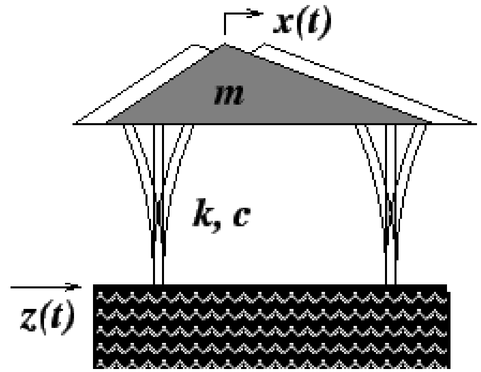
Then the equation of motion for i-th story::

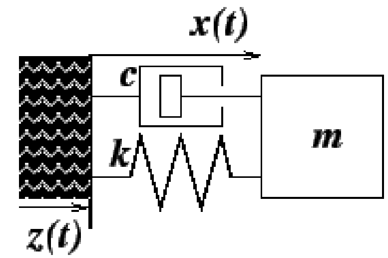
$$m(\ddot{x}_i(t) + \ddot{z}(t)) + kx_i(t) - k(x_{i+1}(t) - x_i(t)) = 0$$

Single-story building oscillation

Construction elements: horizontal plates (rigid masses), walls and beams (flexible and massless).

Then the system behaves as the mass-spring-damper element.





Potential energy stored in walls' elastic deformation and kinetic energy of mass motion:

$$E_n = \frac{1}{2} k x(t)^2 \qquad E_\kappa = \frac{1}{2} m \dot{x}(t)^2$$

By equating the maximal values of potential and kinetic energies, the formula for eigenfrequency can be obtained:

$$\omega = \sqrt{\frac{k}{m}}$$

Multiple-stories building oscillation

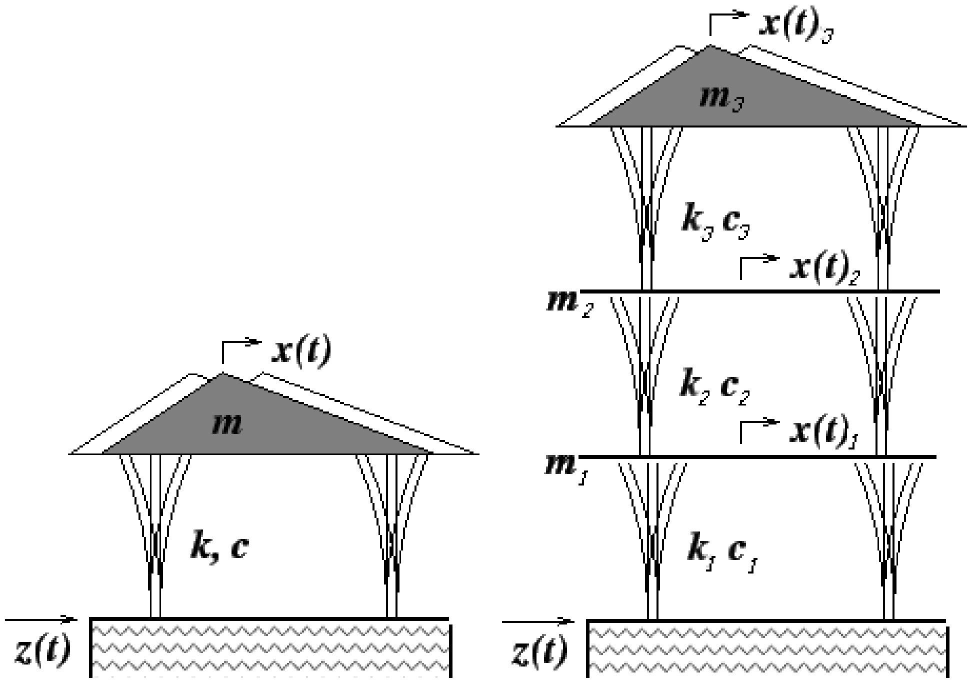
Walls' elasticities $\boldsymbol{k_i}$ and horizontal plates' massses $\boldsymbol{m_i}$ can be written in elasticity and mass matrices $\boldsymbol{K}$ and $\boldsymbol{M}$. Shifts $\boldsymbol{x_i}$ can be written as shift vector $\boldsymbol{X}$. Then expressions for multiple-story building energies can be obtained:

Single Story	Multiple Stories
$PE = \frac{1}{2}kx(t)^2$	$\frac{1}{2}\mathbf{x}^T\mathbf{K}\mathbf{x}$
$KE = \frac{1}{2}m\dot{x}(t)^2$	$\frac{1}{2}\dot{\mathbf{x}}^T\mathbf{M}\dot{\mathbf{x}}$

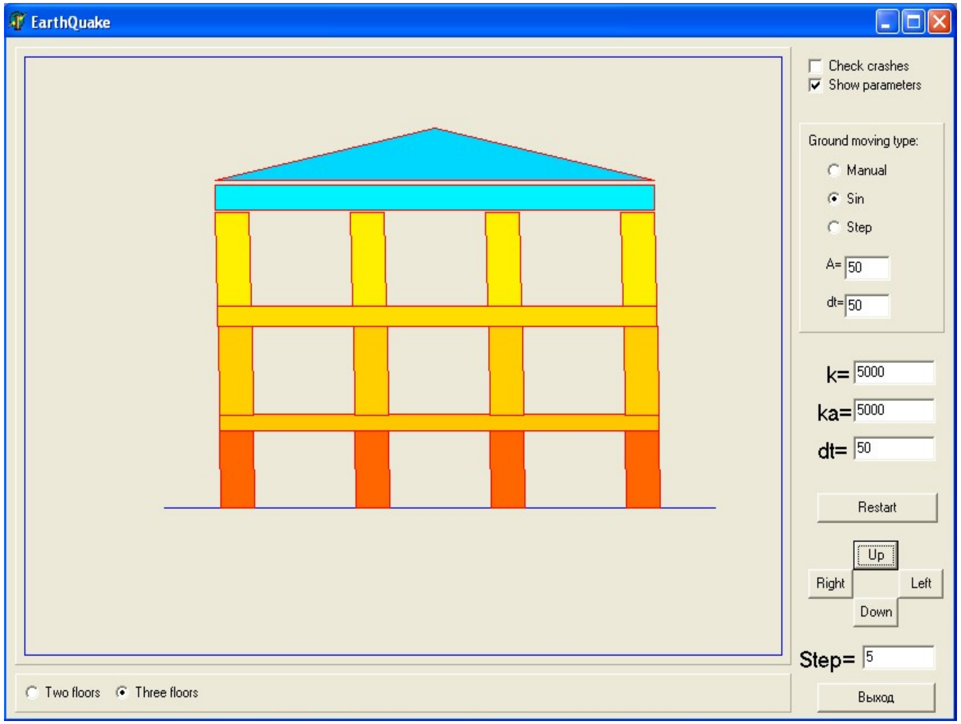
By equating maximal values of energies:

$$K\vec{x} = \omega^2 M\vec{x}$$

which is the eigenvalue problem



Computer model of stresses in oscillating building

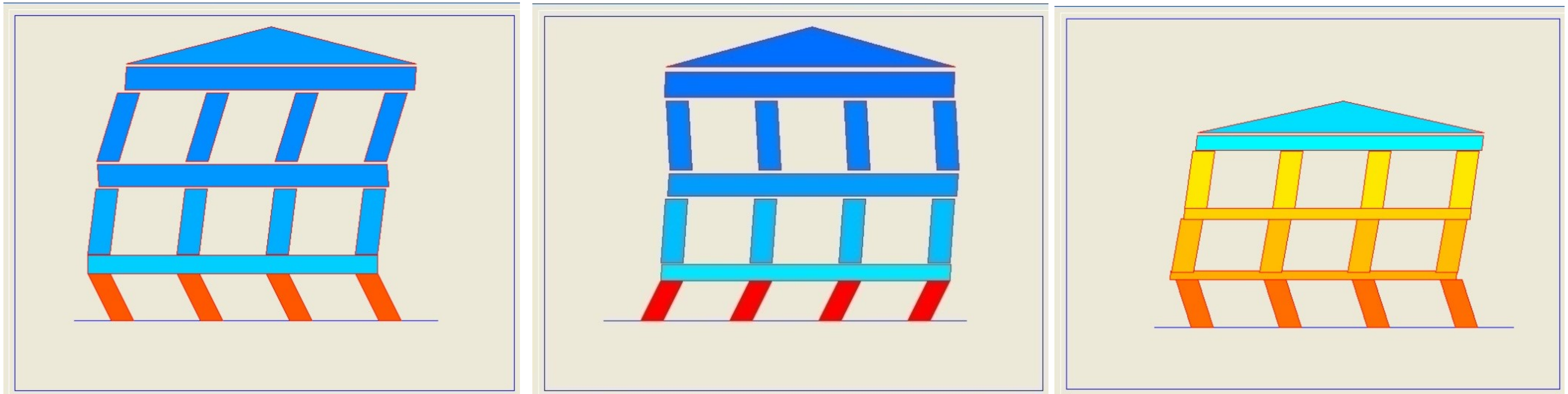


launch

We developed a computer model, which allowed us to describe the occurring stresses quantitatively.

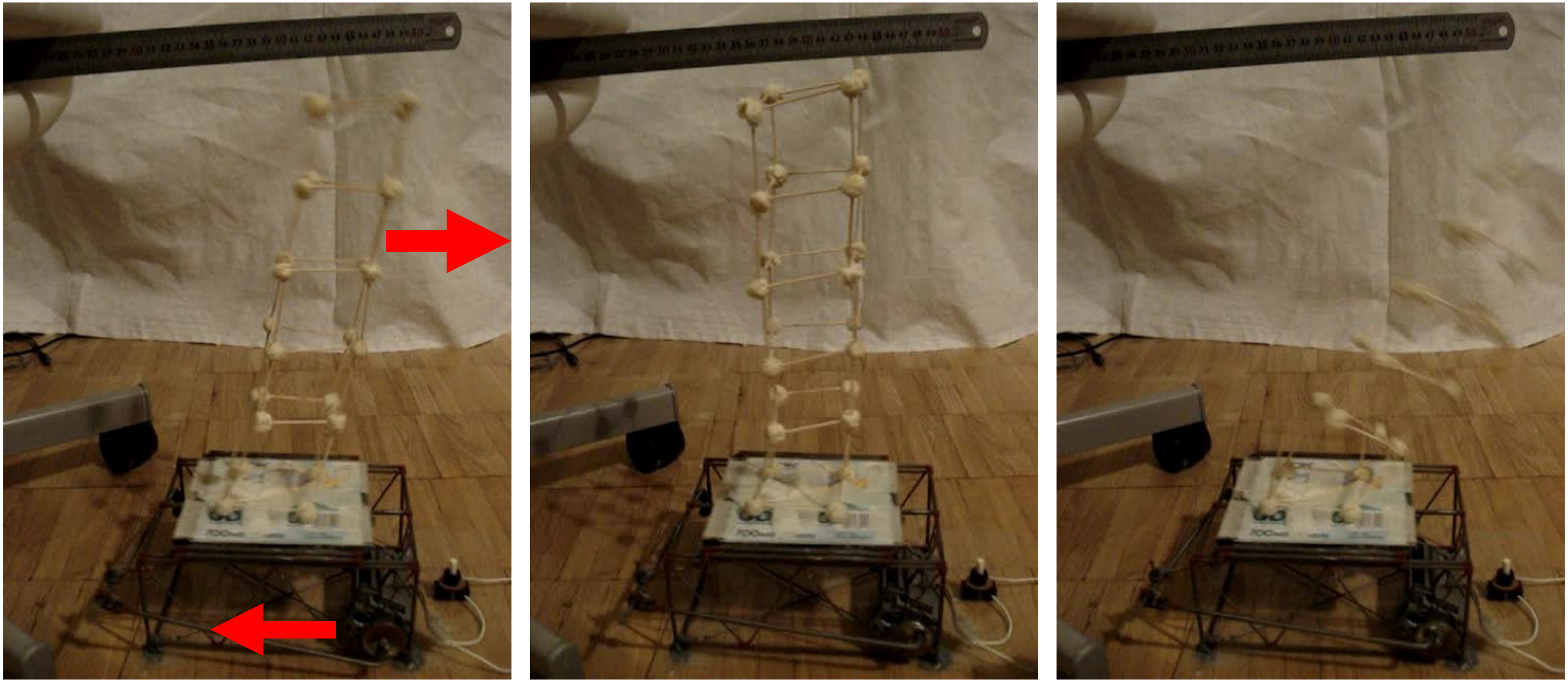
Variable parameters are: structure rigidity, direction, amplitude and frequency of basement shift.

Computer model of stresses in oscillating building



First story is subjected to higher stress, that’s why the building falls aside.

Building failure under non-sine oscillation and with weak ground story: the building is falling aside



Single ground shift, non-sine oscillations

Single ground shift often occurs during earthquakes. In such a situation first stories are moving with basement and the building is falling aside.

Film1

Ground acceleration 0,25±0,2 m/s^2
shear stress 0,7±0,05 Pa

Conclusion: ground story needs to be strengthened and be mobile relatively to basement.

Buildings failure is often avalanche-like. Why?

Buildings have very low density: ≈ 60 kg/m^3 (density of brick ≈ 2800 kg/m^3). If the failure appears in any part of the building, the avalanche-like destruction of all structure starts.

Light metal frame is seismic-resistant!

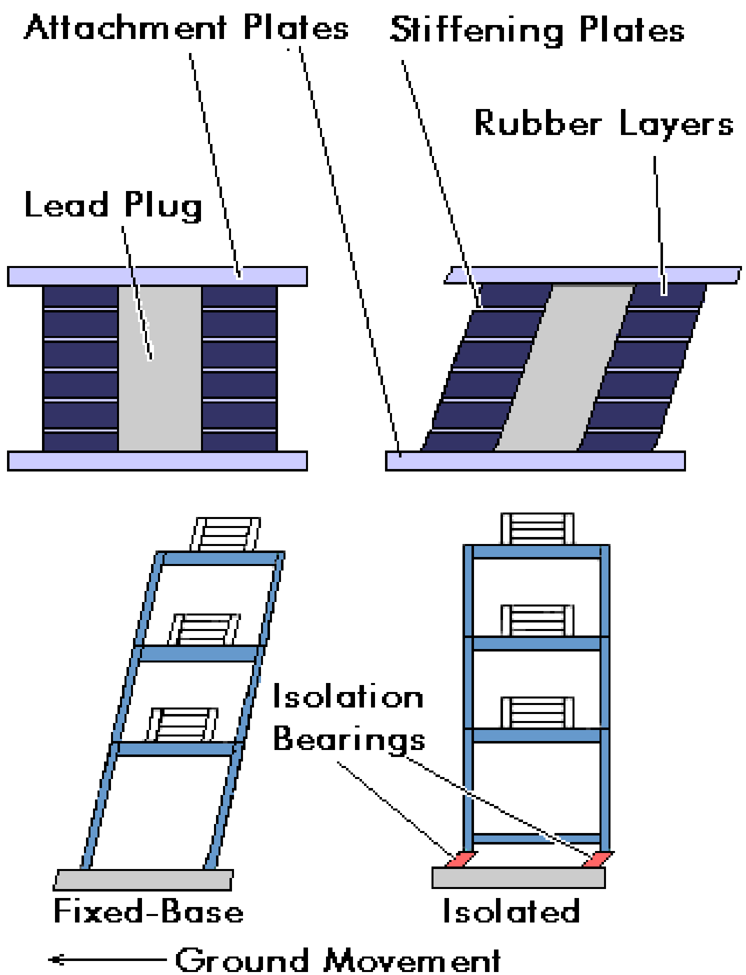
Metal frame is more seismic-resistant than other building elements

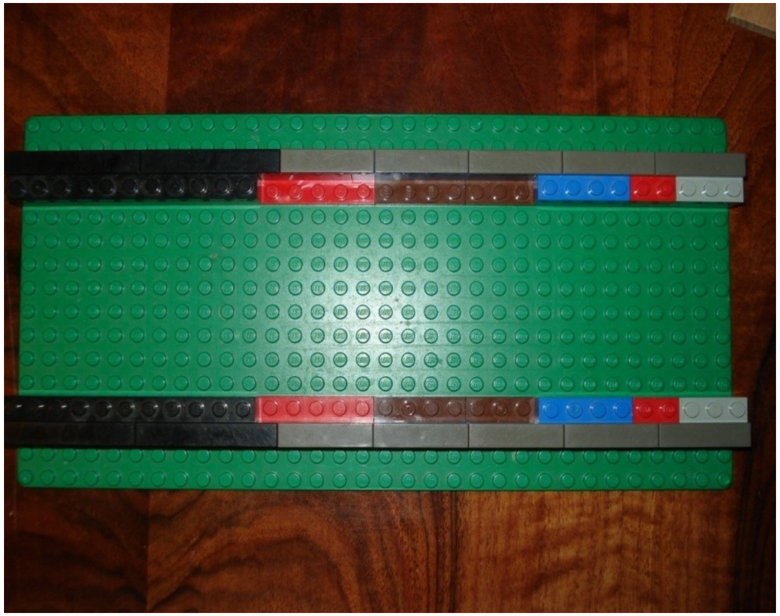
Izmit, Turkey, 17th august, 1999

www.nationalgeographic.com\forcesofnature\earthquake\

Elastic frame

Methods of seismic protection: “isolated” basement





During horizontal ground shift the building stays vertical and keeps its shape.

Two-layer isolated basement made of “Lego™” constructor.

Influence of isolated basement on seismic-resistance

MOV01851.MPG

Building without isolated basement. 10 sec of oscillation

Building with isolated basement. 10 sec of oscillation

The results

- The main causes of building destruction and behavior of different building types were described theoretically and experimentally;
- The main physical values describing inner stresses, deformations, oscillations and failures were researched;
- 4 (four) original experimental setups able to model earthquakes with defined parameters were developed;
- Dozens of building models with different properties: with different height, made of different materials (different fragility, density, elasticity), different eigenfrequencies, different quality factors;
- The parameters influencing on seismic-resistance were researched: height, fragility, elasticity, plasticity, quality factor, eigenfrequency, density distribution;
- Similarity of model and reality was analyzed (theory of similarity and similarity criteria);
- Methods of building seismic-resistance development were suggested and researched:
- Resonance, resistance to different modes of oscillations, difference of deformations under sine and non-sine oscillations, deformation waves propagation with different geometrical properties, amplitudes and frequencies.
- The universal computer model was developed. It models all mentioned parameters and precisely predict the behavior and properties of buildings

Conclusions

- The seismic resistant building needs to:
  - have elastic metal/monolith frame
  - be symmetrical relatively to the center of masses
  - have reinforced stories
  - have uniform density distribution
  - have eigenfrequency different from probable ground oscillation frequency
  - have isolated basement, cross-bracing, dampers
  - have structure elements mobile relatively to each other
  - have ground floor mobile relatively to basement
  - have mobile mass-damper which can dynamically change the building natural frequency.

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Acknowledgements

- Dept. of Physics, St Petersburg State University
- Institute for Physics, St Petersburg State University