

# The QED theory of the multiphoton transitions with cascades

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An ambiguity of separation of cascades from "pure" two-photon decay is confirmed with accurate numerical calculations in a gauge-invariant way. A direct evaluation of two-photon decay width of excited states in H-like ions via imaginary part of two-loop self-energy is presented. We demonstrate that there is fundamental difference between the level width and transition probability in the presence of cascades. The two-photon widths are shown to have no direct connection to the two-photon decay rates for the transitions including cascades.

During the last decade the multiphoton transitions became of high interest for astrophysics. This interest was triggered by the accurate measurements of the temperature and polarization distribution of the Cosmic Microwave Background (CMB) [1], [2]. The launching of the Planck Surveyor enables to perform the measurements with high accuracy. It is a challenge to the theory to perform the calculations of the properties of CMB with the same accuracy. The CMB was formed in the epoch of the cosmological hydrogen recombination and an accurate theory of this formation is therefore required. The modern theory of the cosmological recombination starts from works by Zel'dovich, Kurt and Sunyaev [3] and Peebles [4]. According to [3], [4] the one-photon transition from the upper hydrogen levels to the lower ones did not permit the electrons to recombine, i.e. to reach the ground state. Each photon released in the one-photon transition in one atom was immediately absorbed by another atom. This reabsorption did not allow the radiation to escape the interaction with the matter and after redshifting to form CMB. However if an atom happens to arrive in  $2s$  state then it decays via two-photon transition. Frequencies of these two photons do not fit any particular absorption line. Therefore these two photons escape the reabsorption and the recombination occurs. This was first established in [3], [4] where  $2s \rightarrow 1s + 2\gamma(E1)$  transition was found to be the main channel for the radiation escape and formation of CMB. Hence the recent properties of the CMB are defined by the two-photon processes during the cosmological recombination epoch.

Apart from  $2s \rightarrow 1s + 2\gamma(E1)$  transition as it was noted recently in [5], [6] the two-photon decays from the excited states with the principal quantum numbers  $n > 2$  also can contribute to the radiation escape of the 1% level of accuracy. This idea was further developed and intensively discussed in [7]-[9]. There is a difference between decay of  $ns$  ( $n > 2$ ) and  $nd$  states and decay of  $2s$  state. This difference is due to the presence of cascade transitions as the dominant decay channels in case of  $ns$  ( $n > 2$ ) and  $nd$  levels. For the  $2s$  level the cascades are absent. The cascade photons were assumed to be effectively reabsorbed and therefore the problem of separation "pure" two-photon emission from the cascade photons arises in connection with the escape probability.

A cascade separation problem appeared to be nontrivial. For the first time this question was raised in [10] for the two-photon transitions in the Highly Charged Ions (HCI). The same problem was considered later in [11]. In [12], [13] a general QED approach was developed which allowed for the rigorous description of the multiphoton cascade transitions. This approach was based on the F. Low theory [14] of the spectral line profile in QED. In [12], [13] it was demonstrated that the separation of the cascade contribution from "pure" two-photon decay rate can not be done in an unambiguous way. While in [12] this problem was studied for the HCI, in [15], [16] it was discussed in connection with the cosmological recombination. As an example the two-photon decay of the level  $3s$  in hydrogen was considered:  $3s \rightarrow 1s + 2\gamma(E1)$ . The ambiguity of separation of this cascade was demonstrated numerically: the result was dependent on the method of separation. In this work we reconsider the separation problem with the same example. On a basis of more accurate numerical calculations we are able to show that while the contribution of "pure" two-photon and interference terms vary essentially with the method of cascade separation, the total two-photon transition rate remains invariant. The latter circumstance was not obvious from the results in [15] (because of the poorer accuracy of the numerical calculations). Moreover in this work we prove the gauge invariance of our results by a direct numerical check.

In the second part of our work we investigate the problem of the evaluation of the two-photon level width via the imaginary part of two-loop electron self-energy. This evaluation was performed in a series of works [17]-[20] and it was claimed that the results obtained present an unambiguous way to evaluate the "pure" two-photon contributions to the two-photon transitions with cascades. The derivations were made non-relativistically. The numerical values differ essentially from the ones in [15]. In the present work we rederived the imaginary part of the two-loop contribution in a fully relativistic way and confirmed exactly the results [17]-[19]. However we argue that these results have no direct connection to the two-photon decay rates for the transitions with cascades. The numbers obtained in [17]-[19] and confirmed in our present work are the "pure" two-photon contributions to the level widths which are always smaller than the one-photon contributions. They represent small corrections to the total level widths.

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## List of publications

1. T. Zaliutdinov, D. Solovyev and L. Labzowsky QED model of the radiation escape from matter // *Journal of Physics B: Atomic, Molecular and Optical Physics*, 2012. Vol. 45, P. 165006-1 165006-14
2. T. Zaliutdinov, Yu. Baukina, D. Solovyev and L. Labzowsky Theory of the multiphoton cascade transitions with two photon links: comparison of Quantum Electrodynamical and Quantum Mechanical approaches <http://arxiv.org/abs/1204.2369>
3. *Optics and Spectroscopy*, 2011, Vol. 110, No. 3, pp. 328334. Pleiades Publishing, Ltd., 2011. Original Russian Text T.A. Zalyalyutdinov, D.A. Solovyev, L.N. Labzovskii, 2011, published in *Optika i Spektroskopiya*, 2011, Vol. 110, No. 3, pp. 362368.

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